Scalable Strategies for Stochastic Network Problems

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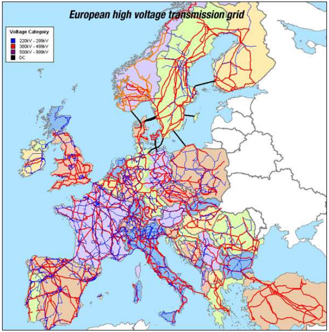
Motivation

How to exploit structure in power grid problems?

- What tools?
  1) Interior Point Methods
  2) Parallel Linear Algebra
- Applications:
  1) AC-SCOPF
  2) DC-OPF



Interior Point Methods (IPM)

Nonlinear Program

$$\min \mathbf{f}(x) \qquad \text{s.t.} \quad \begin{aligned} \mathbf{c}(x) &= 0 \\ x &\geq 0 \end{aligned} \qquad \text{(NLP)}$$

KKT Conditions

$$\begin{aligned} \nabla \mathbf{f}(x) - \nabla \mathbf{c}(x)\lambda - s &= 0 \\ \nabla \mathbf{c}^\top x &= 0 \\ XSe &= 0 \\ x, s &\geq 0 \end{aligned} \qquad \text{(KKT)}$$

$X = \mathrm{diag}(x), S = \mathrm{diag}(s)$

Interior Point Methods (IPM)

Barrier Problem

$$\min \mathbf{f}(x) - \mu \sum \ln x_i \quad \text{s.t.} \quad \begin{aligned} \mathbf{c}(x) &= 0 \\ x &\geq 0 \end{aligned} \qquad (\mathrm{NLP}_\mu)$$

KKT Conditions

$$\begin{aligned} \nabla \mathbf{f}(x) - \nabla \mathbf{c}(x)\lambda - s &= 0 \\ \nabla \mathbf{c}^\top x &= 0 \\ XSe &= \mu e \\ x, s &\geq 0 \end{aligned} \qquad (\mathrm{KKT}_\mu)$$

$X = \mathrm{diag}(x), S = \mathrm{diag}(s)$

- Introduce logarithmic barriers for $x \geq 0$
- For $\mu \to 0$ solution of (NLP_μ) converges to solution of (NLP)
- System (KKT_μ) can be solved by Newton's Method

Newton-Step in IPM

Newton-Step: Augmented System(IPM)

$$\begin{bmatrix} -H - \Theta & \mathcal{A}^\top \\ \mathcal{A} & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} \xi_c - X^{-1} r_{xs} \\ \xi_b \end{bmatrix}$$

where $\Theta = X^{-1}S$, $\quad X = diag(x)$, $\quad S = diag(s)$. Matrix $\mathcal{A}$ is the constraint Jacobian, and H is the Hessian of the Lagrangian function L.

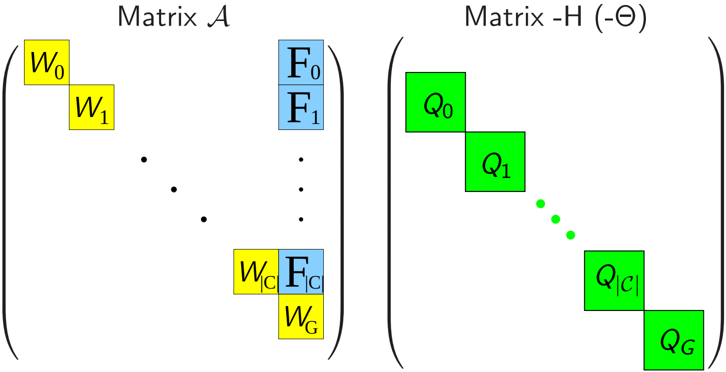
- NLP needs more work to ensure global convergence.
- IPM with filter technique (IPOPT[1]).

[1] Andreas Wächter and Lorenz T. Biegler. "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming". In: *Math. Program.* 106.1, Ser. A (2006), pp. 25–57. ISSN: 0025-5610.

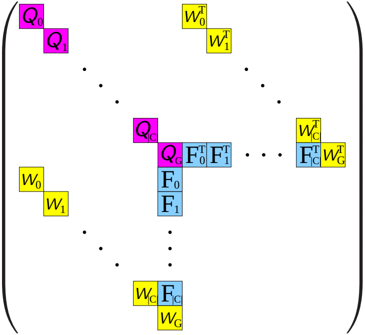
Parallel Linear Algebra for IPM

Newton-Step: Augmented System(IPM)

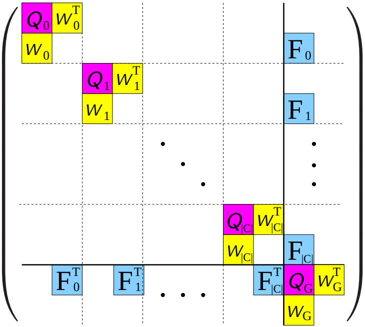
$$\Phi = \begin{bmatrix} -H - \Theta & \mathcal{A}^\top \\ \mathcal{A} & 0 \end{bmatrix} \tag{1}$$



Structures of $\mathcal{A}$, Q and Φ:

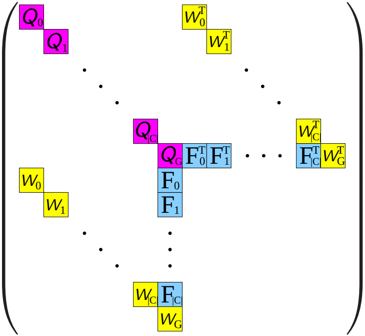


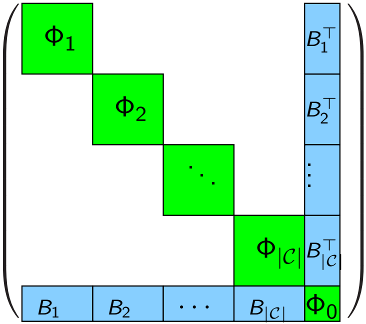
$$\begin{pmatrix} Q & \mathcal{A}^\top \\ \mathcal{A} & 0 \end{pmatrix}$$



$$P \begin{pmatrix} Q & \mathcal{A}^\top \\ \mathcal{A} & 0 \end{pmatrix} P^{-1}$$

Structures of $\mathcal{A}$, Q and Φ:





$$\begin{pmatrix} Q & \mathcal{A}^\top \\ \mathcal{A} & 0 \end{pmatrix} \qquad P \begin{pmatrix} Q & \mathcal{A}^\top \\ \mathcal{A} & 0 \end{pmatrix} P^{-1}$$

Bordered block-diagonal structure in Augmented System!

Exploiting Structure in IPM

Block-Factorization of Augmented System Matrix

$$\underbrace{\begin{pmatrix} \Phi_1 & & & B_1^\top \\ & \ddots & & \vdots \\ & & \Phi_n & B_n^\top \\ B_1 & \cdots & B_n & \Phi_0 \end{pmatrix}}_{\Phi} \underbrace{\begin{pmatrix} x_1 \\ \vdots \\ x_n \\ x_0 \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_n \\ \mathbf{b}_0 \end{pmatrix}}_{\mathbf{b}}$$

Solution of Block-system by Schur-complement

The solution to $\Phi x = \mathbf{b}$ is

$$\begin{aligned} x_0 &= C^{-1}\mathbf{b}_0, \quad \mathbf{b}_0 = b_0 - \textstyle\sum_i B_i \Phi_i^{-1} \mathbf{b}_i \\ x_i &= \Phi_i^{-1}(\mathbf{b}_i - B_i^\top x_0), \qquad i = 1, \dots, n \end{aligned}$$

where C is the *Schur-complement*

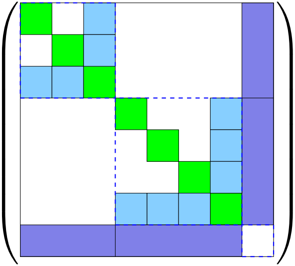
$$C = \Phi_0 - \sum_{i=1}^{n} B_i \Phi_i^{-1} B_i^\top$$

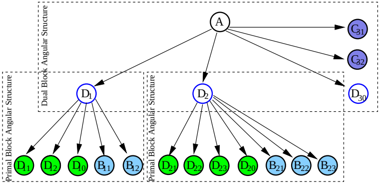
$\Rightarrow$ only need to factor Φ_i, not Φ

Paraller Linear Algebra for the Structured Problem

Parallel IPM Implementation

- Jacek Gondzio and Andreas Grothey: Exploiting structure in parallel implementation of interior point methods for optimization. (OOPS)
- Cosmin G. Petra and Mihai Anitescu: A preconditioning technique for Schur complement systems arising in stochastic optimization.





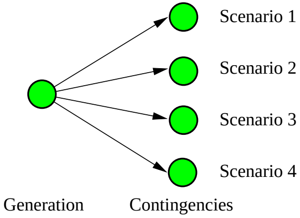
Paraller Linear Algebra for the Structured Problem

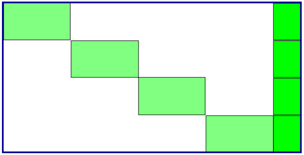
Structure comes from ...

- Robust Stochastic Programming (scenarios)
- Network (partitions)

- Still computationally expensive: memory and communication
- Possible remedies:
  a) scenario elimination
  b) iterative method (for solving linear system)

Scenario Elimination





Scenario Elimination

- Start from a smaller model with one "base" scenario.
- Generate a central point for the reduced problem.
- Fix the global variables and find feasible solutions of other scenarios. (Scenario Analysis)
- Add violated scenario dynamically.

AC-SCOPF: Scenario & Scenario Elimination

Generic AC OPF Model

Optimal Power Flow (OPF)

A minimum cost power generation model.

Parameters

α_l, β_l	conductance and susceptance of line l
β_b	susceptance of power source at bus b
d_b^P, d_b^Q	real and reactive power demand at bus b
f_l^+	flow limit for line l

Variables

v_b	Voltage level at bus b
δ_b	Phase angle at bus b
$p_g,\ q_g$	Real and reactive power output at generator g
$f_{(i,j)}^P, f_{(i,j)}^Q$	Real and reactive power flow on line $l = (i,j)$

Generic AC OPF Model

Constraints

- Kirchhoff's Voltage Law (KVL)

$$
\begin{aligned}
f^P_{(i,j)} &= \alpha_l v_i^2 - v_i v_j[\alpha_l \cos(\delta_i - \delta_j) + \beta_l \sin(\delta_i - \delta_j)] \\
f^Q_{(i,j)} &= -\beta_l v_i^2 - v_i v_j[\alpha_l \sin(\delta_i - \delta_j) - \beta_l \cos(\delta_i - \delta_j)]
\end{aligned}
$$

- Kirchhoff's Current Law (KCL)

$$
\begin{aligned}
\sum_{g|o_g=b} p_g &= \sum_{(b,i)\in L} f^P_{(b,i)} + d^P_b, \quad \forall b \in \mathcal{B} \\
\sum_{g|o_g=b} q_g - \beta_b v_b^2 &= \sum_{(b,i)\in L} f^Q_{(b,i)} + d^Q_b, \quad \forall b \in \mathcal{B}
\end{aligned}
$$

- Line Flow Limits at both ends of each line

$$
\begin{aligned}
(f^P_{(i,j)})^2 + (f^Q_{(i,j)})^2 &\le (f_l^+)^2 \\
(f^P_{(j,i)})^2 + (f^Q_{(j,i)})^2 &\le (f_l^+)^2
\end{aligned}
$$

- Reference bus

$$\delta_0 = 0$$

$\Rightarrow$ AC OPF is a nonlinear programming problem

Security-Constrained Optimal Power Flow (SCOPF)

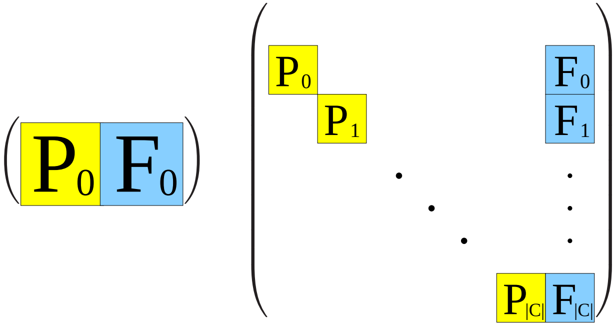
(N-1)SCOPF

Network should survive the failure of any one line (possibly after limited corrective actions) without line-overloads.

Setup

- Contingency scenarios $c \in \mathcal{C}$, each has its own power transmission network.
- Real generation p_g and Voltage v_g at the PV bus keep same for all contingencies. (Global Variables)
- Each contingency has its flow, voltage, phase angle and reactive generation: $f_c^{P/Q}, v_c, \delta_c, q_c$. (Local Variables)
- Possible modification of generator output p_c in each contingency scenario.
- Seek a generator setting that does not create line overloads for any contingency

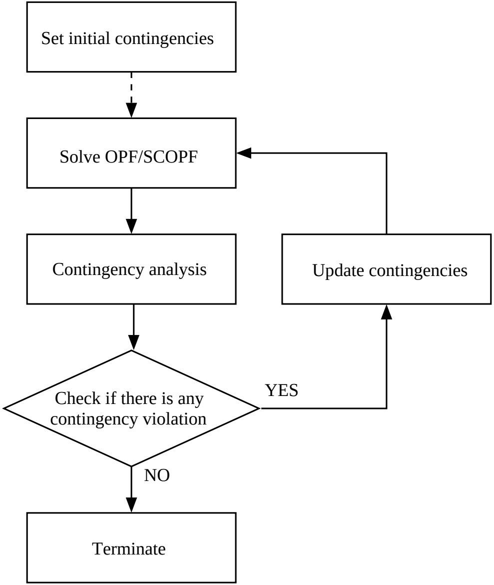
Structure of the Problem



OPF SCOPF

- SCOPF (like many other structured problems) consists of a small core that is repeated many times.
- "n-1" requires the inclusion of many contingency scenarios.
- Only a few contingencies are critical for operation of the system (but which ones)?

Flow Chart for Solving SCOPF: State of the Art



Structured IPM with Scenario Elimination

We do:

- Start from solving much smaller problem of same structure, as the practical SCOPF solution technique.
- Apply contingency analysis between IPM steps.

Advantages:

- Total number of linear algebra to build Schur complement in each IPM iteration is proportional to the size of scenarios.
- Combine two iterative processes (IPM and the pratical way to solve SCOPF) in one. $\rightarrow$ only one outter loop!

Numerical Result: OOPS

		Original			Scenario Elimination		
Prob	No.Sce	time(s)	iters	No.Act	time(s)	iters	No.ActSce
A	1	<0.1	9	0	<0.1	9	1
B	2	<0.1	22	0	<0.1	9	1
6	2	<0.1	13	2	<0.1	13	2
IEEE_24	38	5.7	41	6	3.9	30	6
IEEE_48	78	51.8	71	11	32.4	52	15
IEEE_73	117	204.1	97	16	156.7	92	25
IEEE_96	158	351.5	106	20	252.9	76	27
IEEE_118	178	???	??	42	1225.2	75	46
IEEE_192	318	2393.7	132	26	1586.0	92	40
L26	41	0.4	14	2	0.3	11	2
L200	371	264.3	53	7	56.4	25	7
L300	566	1153.1	88	17	196.3	22	20

Table: Scenario elimination results

- More than 200% computational resources are saved!

DC-OPF: Network Partition

Another Scalable Strategy for Parallelism

Idea: Decompose the model by the power system behavior

- Graph partitioning technique.
- Decompose the large network into several "equal-sized" pieces.
- Minimize the number of edge cuts between separated components.

- Advantages: Solve the model for each piece of cake in parallel!
- Difficulties: Unusual as generic stochastic programming: Partitioning may introduce high degree of coupling vars and constraints.

DC-OPF formulation

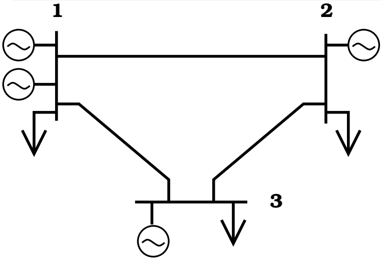
DC-OPF formulation (Default)

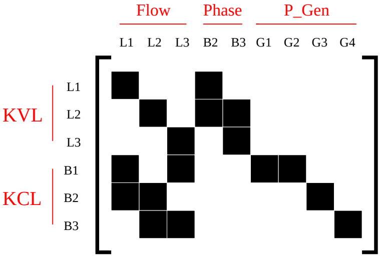
- Kirchhoff's Voltage Law

$$f_l^P = -\frac{v^2}{r_l}\sum_{b\in\mathcal{B}} a_{bl}\delta_b, \quad \forall l \in \mathcal{L}$$

- Kirchhoff's Current Law

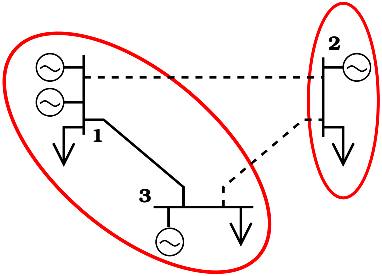
$$\sum_{g|o_g=b} p_g = \sum_{(b,i)\in\mathcal{L}} f_{(b,i)}^P + d_b^P, \quad \forall b \in \mathcal{B}$$

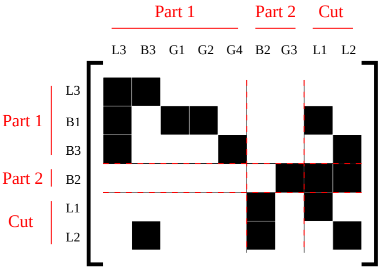




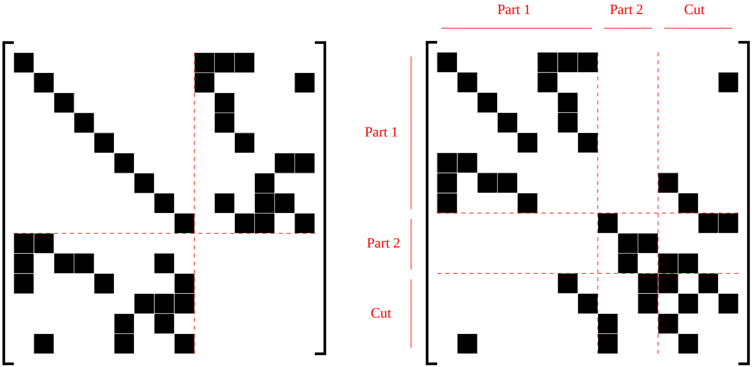
The structure of the matrix components in IPM

- Each partition corresponds to a diagonal block in the constraint Jacobian.
- Variables and constraints corresponding to the cuts are moved to the borders.





Structures of the Augmented System:



Augmented System

Reordered Augmented System

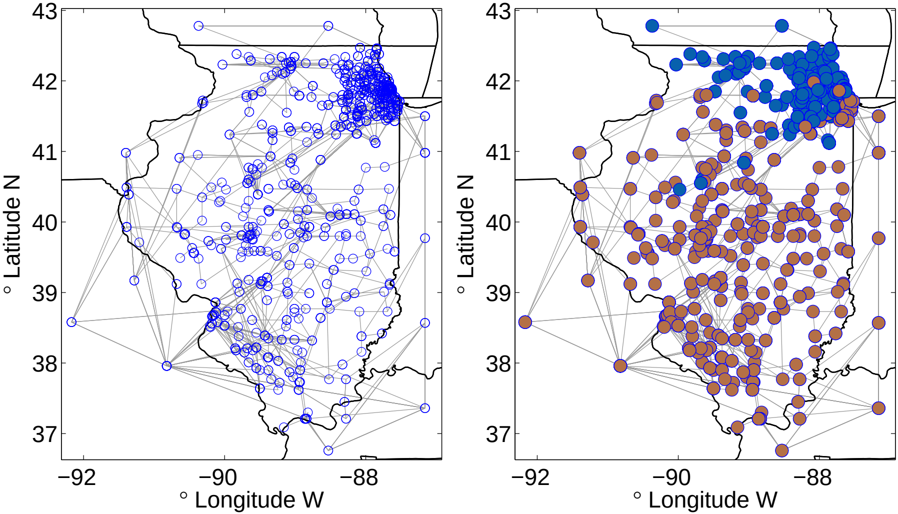
- The size of Schur complement is 2 times #.cuts!

The Illinois system

How does the network partition look like for the real system?

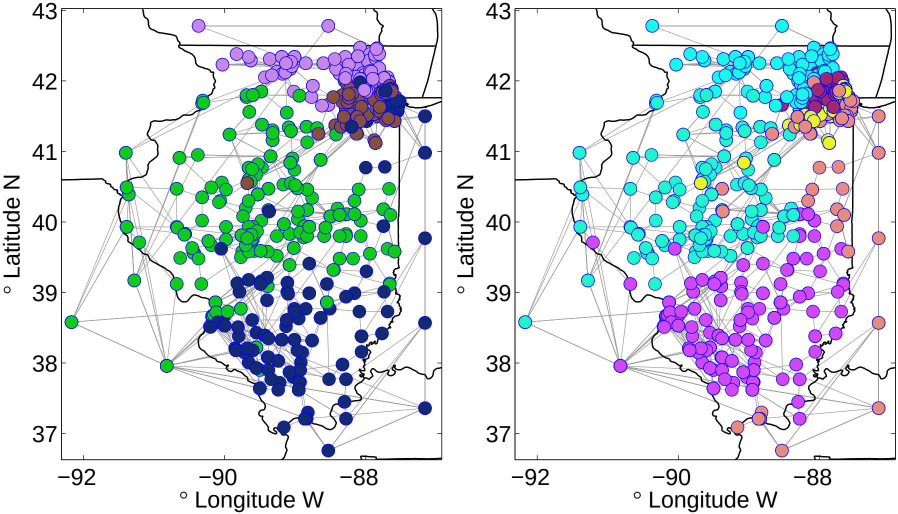
- Illinois system: 1908 buses and 2522 lines
- Is network partition obvious?
- How many coupling variables and constraints will be introduced?
- How would this affect the computational scalability?
- What number of partitions is sensible to apply?

The Illinois system



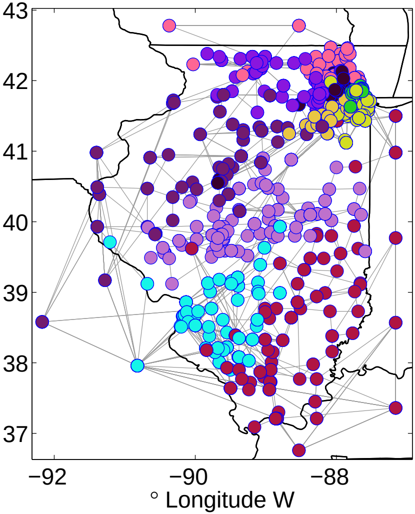
Illinois system and the system with 2 partitions.

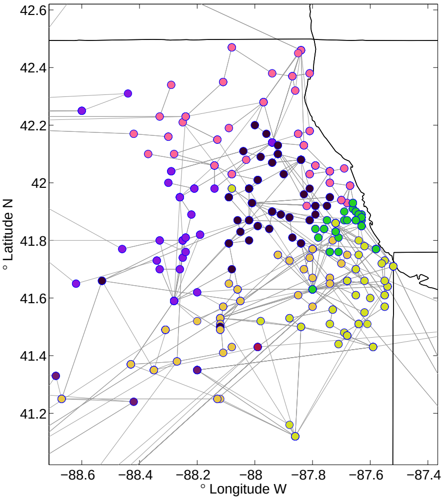
The Illinois system



Illinois system with 4 and 6 partitions.

The Illinois system



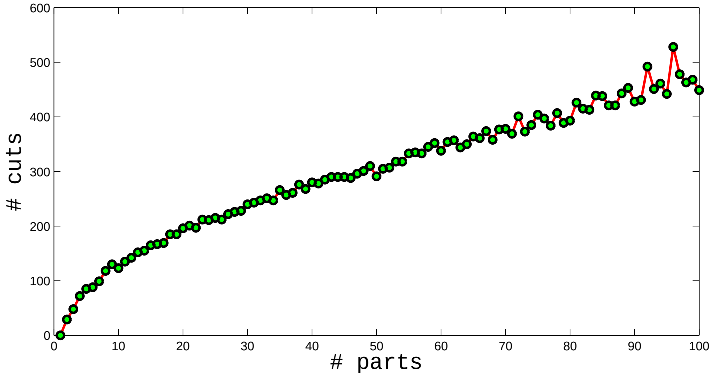


Illinois system with 10 partitions.

The Illinois system

Coupling constraints = # of the edge-cuts

- Determine the size of the Schur complement. ($2\times$)
- Communication between processes! Parallel efficiency!



Numerical results from prototype

- 4690 variables and 4430 constraints $\rightarrow$ one time slot

Illinois system: DC-OPF

- Network partition:
  less than 0.1s for network partitions (by Metis), regardless of the number of partitions (from 1 to 100).
  Each part only contains 20 buses (with 100 partitions)!
- Solution time:
  4 partitions with four processes (72 cuts):
  a) faster than solving the problem in serial.
  100 partitions with four processes (449 cuts):
  b) slower than solving the problem in serial. (Only 191 buses in each part, but the size of Schur complement is large $\rightarrow$ more expensive to solve this problem.)

Scenarios can also be included in the model $\rightarrow$ nested structure.

Conclusions

Problems is complicated

- Illinois system with 24 hours slots and Wind:
  10 mins in serial (CPLEX) for the relaxation of the Unit Commitment.i

We expect:

- 24 time steps UC for the Illinois system: $\approx$ 2.5 mins in parallel
- Partitioning with 10 parts is appliable: 100 cuts per time slot; 100*24 $=$ 2400 Coupling variables for the full problem; speed up the solution time by a factor of 10!

Future Work: merge all the tools

- Complete the NLP tool to solve the AC stochastic problem (Time! UC/ED! Security!)
- Apply the scenario elimination technique and iterative methods.

Conclusions

- Thank you for your attention!